Unit 13: PROBABILITY

This unit will show you how to:

- Understand the concepts of random experiment, outcome and event
- Understand the concepts of experimental and theoretical probability
- Calculate the probability of an event
- Calculate the probability of mutually exclusive events
- Use two-way tables to calculate probabilities

Keywords	
Random experiment	Sure event
Outcome	Impossible event
Sample space	Mutually exclusive events
Elementary event	Probability
Event	Two-way table

13.1.- RANDOM EXPERIMENTS. OUTCOMES AND EVENTS

Random experiments are those that can't be predicted with total certainty.

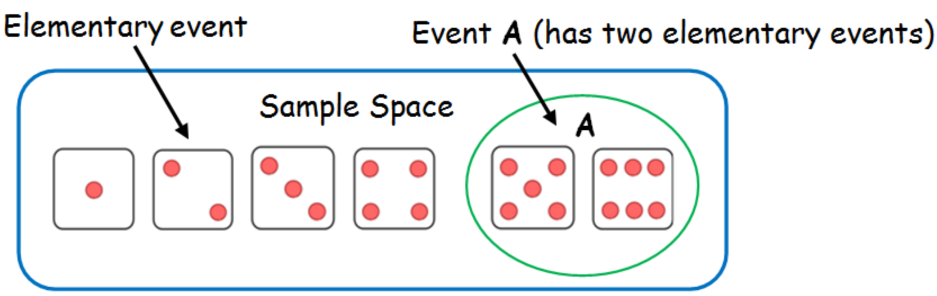
Examples: tossing a coin, rolling a dice, choosing a card from a deck, choosing a marble from a bag, …

The set of all possible **outcomes** (results) of a random experiment is called the **sample space** (S).

The basic outcomes of an experiment are also known as **elementary events**.

An **event** is a set of outcomes, that is, a subset of the sample space.

Example: rolling a dice



Sample Space: $S = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

$\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$ and $\{6\}$ are elementary events

$A = \{5,\ 6\}$ is an event that has two elementary events

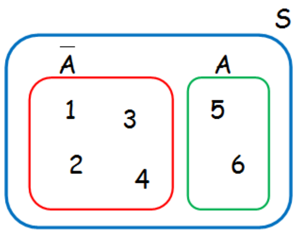
Three important events

Sure event: event that always occurs, whatever the result of the experiment is. The sample space S is a sure event.

Impossible event: event that never occurs, whatever the result of the experiment is. The empty set $\varnothing$ is an impossible event.

Complement: the complement of an event A, denoted by $\overline{A}$, is the set of all basic outcomes in the sample space that do not belong to A.

Example: For the experiment of rolling a dice,
if $A = \{5,\ 6\}$, then $\overline{A} = \{1,\ 2,\ 3,\ 4,\ 5\}$



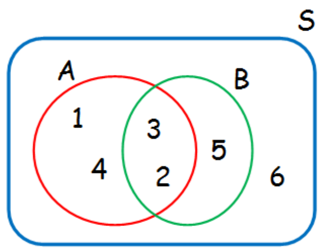
Union and intersection of events

If A and B are two events in a sample space S,

- $A \cup B$ = "either A or B will occur", that is, $A \cup B$ is the set of all outcomes in S that belong to either A or B.
- $A \cap B$ = "both A and B will occur", that is, $A \cap B$ is the set of all outcomes in S that belong to both A and B.

Example: $A = \{1,\ 2,\ 3,\ 4\}$ $\quad B = \{2,\ 3,\ 5\}$

$A \cup B = \{1,\ 2,\ 3,\ 4,\ 5\}$ $\quad A \cap B = \{3,\ 2\}$



Mutually exclusive events

A and B are **mutually exclusive events** if they cannot happen at the same time. If A and B are mutually exclusive events, they have no basic outcomes in common, that is, $A \cap B = \varnothing$. Mutually exclusive events are also called **disjoint events** or **incompatible events**.

Example: If we roll a dice, the events A = "getting an even number" and B = "getting an odd number" cannot happen at the same time.

$A = \{2,\ 4,\ 6\}$ $\quad B = \{1,\ 3,\ 5\}$ $\quad A \cap B = \varnothing$

Exercise 1

How many elements are in the sample space of tossing 3 pennies? List the sample space.

Exercise 2

A twelve sided dice with sides numbered 1 through 12 is rolled.

a) Write the sample space for this experiment.
b) Write the events:
   A = "less than 4", B = "odd number", C = "greater than 5"
c) Write the events $A \cup B$, $B \cup C$, $A \cap B$ and $A \cap C$.
d) Are A and C mutually exclusive events?

Exercise 3

The five staid members of the Oxford Mathematical Society are **A**dams, **B**agley, **C**riswell, **D**uggens, and **E**nright. These gentlemen will randomly choose a committee of two to study an old calculus manuscript. Write the sample space for this experiment by listing all possible committees of two.

13.2.- THEORETICAL AND EXPERIMENTAL PROBABILITY

Theoretical probability

You can calculate the **theoretical probability** when an event is fair or unbiased.

For example:

A fair coin is thrown.

Probability of a fair coin landing on 'heads' $= \frac{1}{2}$.

one head, two outcomes

An unbiased dice is rolled.

Probability of an unbiased dice landing on '2' $= \frac{1}{6}$

one '2', six outcomes

Theoretical probability is based on **equally likely outcomes**.

Theoretical probabilities are calculated on the assumption that the number of favourable outcomes and the number of possible outcomes are as expected.

$$\text{Probability of an outcome} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Therefore, probability is measured on a scale from 0 to 1.

0	—	1
Impossible event: cannot happen		Sure event: certain to happen

Example 1: There are 30 students in Class 10Z, 18 girls and 12 boys. All of the students are aged 14 or 15 and all own a mobile phone. A student is chosen at random from Class 10Z. What is the probability that the student chosen

a) is aged 10 b) owns a mobile phone c) is a girl d) is a boy?

a) This outcome cannot happen. All the students are 14 or 15, so $P(\text{aged } 10) = 0$.

b) This outcome is certain to happen. All the students own a mobile phone, so $P(\text{owns mobile}) = 1$.

c) There are 18 girls and a total of 30 students, so $P(\text{girl}) = \dfrac{18}{30}$.

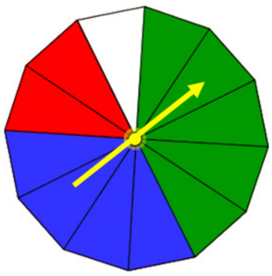
d) There are 12 boys and a total of 30 students, so $P(\text{boy}) = \dfrac{12}{30}$.

The probability that a girl is chosen or that a boy is chosen is certain to happen as the students are either girls or boys.

Notice that $P(\text{girl}) + P(\text{boy}) = \dfrac{18}{30} + \dfrac{12}{30} = \dfrac{30}{30} = 1$

> Sum of probabilities of all possible outcomes $= 1$

Example 2: A spinner has 12 equal sides: five green, four blue, two red and one white. The spinner is spun. What is the probability that the spinner lands on



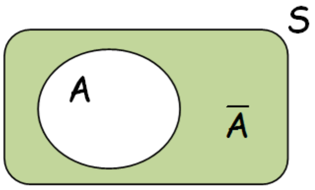
a) green or white b) blue or red or white

a) $P(\text{green or white}) = \dfrac{6}{12} = \dfrac{5}{12} + \dfrac{1}{12} = P(\text{green}) + P(\text{white})$

b) $P(\text{blue or red or white}) = \dfrac{7}{12} = \dfrac{4}{12} + \dfrac{2}{12} + \dfrac{1}{12} = P(\text{blue}) + P(\text{red}) + P(\text{white})$

> If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

Remember that the sample space, S, of a random experiment is a <u>sure event</u>.



The complement of an event A is the same as the event "not A".

$\bar{A} = \text{not } A$

$$1 = P(S) = P(A \cup \bar{A}) = P(A) + P(\bar{A}) \Rightarrow P(\bar{A}) = 1 - P(A)$$

For an event A, $P(\text{not } A) = 1 - P(A)$ or $P(A) = 1 - P(\text{not } A)$

You can use the probability of a particular outcome happening to calculate the **expected number** of times it will occur.

Expected number = Total number of outcomes × Probability of a particular outcome happening

Example 3: A spinner has 8 sides, of which 4 show squares, 3 show triangles, and 1 shows a circle. The spinner is spun 400 times. Find the number of times the spinner is expected to show

a) a square b) a triangle

a) Expected number of squares $= 400 \times P(\text{square}) = 400 \times \frac{4}{8} = 200$

b) Expected number of triangles $= 400 \times P(\text{triangle}) = 400 \times \frac{3}{8} = 150$

Exercise 4

A nine sided dice has one digit (1, 2, 3, 4, 5, 6, 7, 8, 9) on each of its sides. The dice is rolled. What is the probability that the dice lands

a) on the number 7
b) not on the number 7
c) on an odd number
d) on a multiple of 3
e) on a multiple of 5
f) not on a multiple of 5?

Exercise 5

A spinner has circles, squares and triangles on its face. The table gives the probabilities of landing on circle, square and triangle. Work out the value of x.

Outcome	Circle	Square	Triangle
Probability	0.25	0.625	x

Exercise 6

Two fair coins are simultaneously tossed. Find the probability of

a) showing head on the first coin
b) Showing tail on the second coin
c) showing at least one head
d) not showing only tails.

Exercise 7

The table shows information about the type of pet owned by students in Class 10A. No student owns more than one pet.

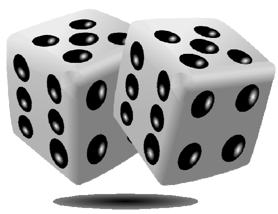
Pet	Cat	Dog	Hamster	Fish	No pet
Number of students	7	8	2	4	9

a) How many students are there in the class?
b) One student is chosen at random from the class. Work out the probability that the student chosen will own

i) a cat
ii) a cat or a dog
iii) a dog or a fish
iv) a cat or a hamster
v) no pets
vi) a pet

Exercise 8

A pair of dice is rolled. Find the probability of



a) showing sum of 6
b) not showing the sum of 10
c) showing the difference of 1
d) showing 6 on the first dice or 4 on the second dice.

Exercise 9

A card is drawn from a Spanish deck of 40 cards. Find the probability of

a) drawing a king
b) drawing either a "bastos card" or a "copas card".

Experimental probability

Experimental probability is also known as **relative frequency**.

Relative frequency is the proportion of successful trials in an experiment.

The more trials that are carried out the more reliable the estimate of probability.

Example: Rachel has a mixed colours packet of seeds. She plants 10 seeds each week for 7 weeks. The table shows the number of purple flowers that grew in each group of seeds.

Week	1	2	3	4	5	6	7
Number of purple flowers	4	6	5	7	4	6	5

a) Work out the relative frequency of a purple flower.
b) Find the best estimate of the probability of getting a purple flower.

a)

Week	1	2	3	4	5	6	7
Number of purple flowers	4	6	5	7	4	6	5
Relative frequency	$\frac{4}{10}$	$\frac{10}{20}$	$\frac{15}{30}$	$\frac{22}{40}$	$\frac{26}{50}$	$\frac{32}{60}$	$\frac{37}{70}$

For each successive week, we find the total number of purple flowers and the total number of seeds planted.

b) The best estimate includes all the results. $P(\text{purple flower}) = \frac{37}{70}$

You can compare the relative frequency (experimental probability) of an outcome with the theoretical probability. If they are quite different, the experiment may be **biased**.

Example: Dan suspects that a particular dice has a bias towards the number 3. Dan rolls the dice 30 times and gets these results

4	3	3	3	6	5	1	3	2	5
1	3	4	5	3	3	6	2	5	4
6	3	1	3	6	5	3	2	4	3

a) What is the relative frequency of rolling a 3?
b) Is the dice biased towards 3? Explain your answer.

a) The number 3 is rolled 11 times. Relative frequency $= \frac{11}{30}$

b) Theoretical probability of rolling a '3' is $P(3) = \frac{1}{6}$

In 30 rolls expected number of 3s $= 30 \times \frac{1}{6} = 5$

5 is not close to 11 so the dice does appear to be biased towards 3.

Exercise 10

A spinner has 10 equal sides, 5 black and 5 red. Dave carries out an experiment. He spins the spinner 280 times. The spinner lands on black 133 times. Is the spinner fair? Explain your answer.

Exercise 11

Clara suspects that a coin is biased. She flips the coin and notes how many heads she gets in each group of 10 flips. Clara flips the coin 100 times in total. The table shows her results.

Group of 10 flips	1	2	3	4	5	6	7	8	9	10
Number of heads	4	3	4	2	5	4	4	3	3	2
Relative frequency										

a) Copy the table and complete for the relative frequency.
b) Write the best estimate of the probability on the coin landing on heads.
c) Is the coin biased? Explain your answer.

13.3.- PROBABILITY IN TWO-WAY TABLES

Probabilities can be found from information given in a **two-way table**.

Example 1: Each of the students in Class 10Z went abroad last year. The two-way table shows some information about the countries visited.

	Europe	**America**	**Rest of the world**	**Total**
Girls	13	3	2	18
Boys	9	2	1	12
Total	22	5	3	**30**

One student is chosen at random from Class 10Z. Write down the probability that the student

a) is a boy who visited America
b) visited America or the Rest of the world.

a) Probability that the student is a boy who visited America $= \frac{2}{30}$

b) P(America or Rest of the world) = P(America) + P(Rest of the world) =

$$= \frac{5}{30} + \frac{3}{30} = \frac{8}{30}$$

Exercise 12

100 students are asked about if they prefer to go swimming or to the gym.

	Swimming	Gym	Total
Boys	25		45
Girls			
Total	64		**100**

a) Complete the table.
b) One student is chosen at random. Find the probability that the student

i) is a boy
ii) prefers go to the gym
iii) is a girl who prefers to go swimming
iv) is not a girl.

Exercise 13

The two-way table shows some information about the preferred subject of the 120 students in Year 10.

	Science	Humanities	Other subjects	Total
Girls	29		32	
Boys		3		
Total	56	21		**120**

a) Copy and complete the table.
b) One student is chosen at random. Find the probability that the student

i) is a girl
ii) prefers Humanities
iii) prefers Humanities or Science
iv) is a boy who prefers Science.

c) This year group is typical of the whole school. There are 600 students in the school. How many students of the whole school would you expect to prefer

i) Science
ii) Humanities?