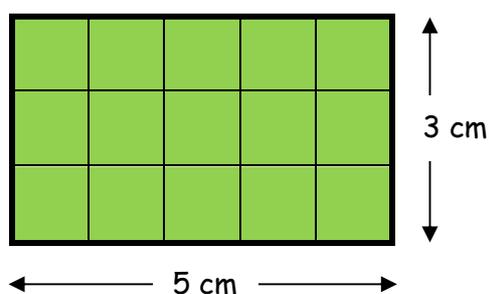


# Unit 12: AREAS AND PERIMETERS

## 12.1.- AREA AND PERIMETER

Look at the following rectangle. Its sides are 5 and 3 cm.



Remember that the **area** of a shape is the amount of space it covers.

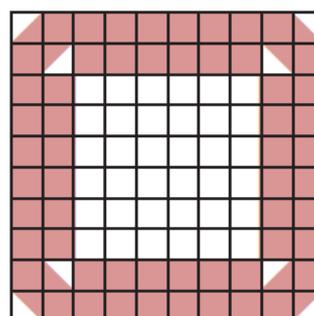
Since a **square centimetre** is the area of a square that is 1 cm on each side, the area of the rectangle above is  $3 \cdot 5 = 15 \text{ cm}^2$ .

The **perimeter** of a plane shape is the total length around it.

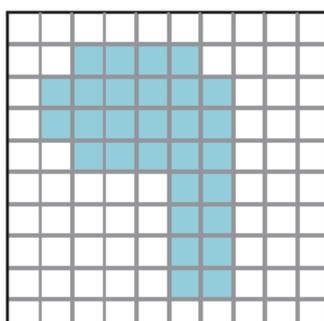
To get the perimeter of the rectangle above, add the lengths of all the sides, that is, is  $3 + 3 + 5 + 5 = 16 \text{ cm}$ .

### Exercise 1:

Determine the area of this picture.  
Each shaded portion represents 1 square unit (1 sq unit).



### Exercise 2:



Determine the area and the perimeter of this picture.  
Each shaded portion represents 1 square unit (1 sq unit).

## 12.2.- AREA AND PERIMETER OF THE QUADRILATERALS

### Square



$s$  = length of side

$$\text{Area} = s^2$$

$$\text{Perimeter} = 4s$$

### Rectangle

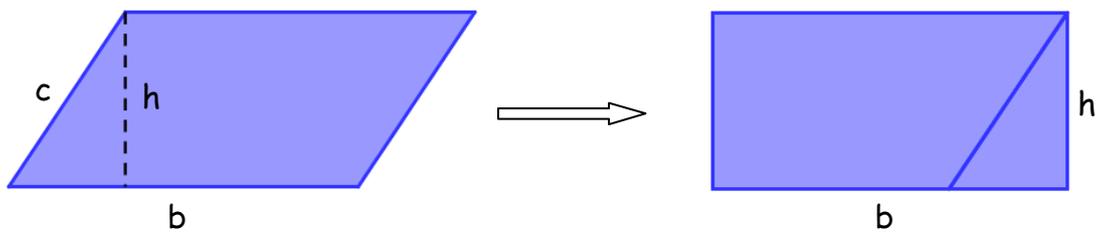


$l$  = length     $w$  = width

$$\text{Area} = l \cdot w$$

$$\text{Perimeter} = 2l + 2w$$

### Parallelogram



If we move the triangle on the left to the right, the area of the rectangle that we get is exactly the same as the area of the parallelogram.

Therefore the area of the parallelogram will be  $A = b \cdot h$ .

$b$  = base     $h$  = height

$$\text{Area} = b \cdot h$$

$$\text{Perimeter} = 2b + 2c$$

### Exercise 3:

A swimming pool was 10 m wide and 8 m long. What is the area of the pool?

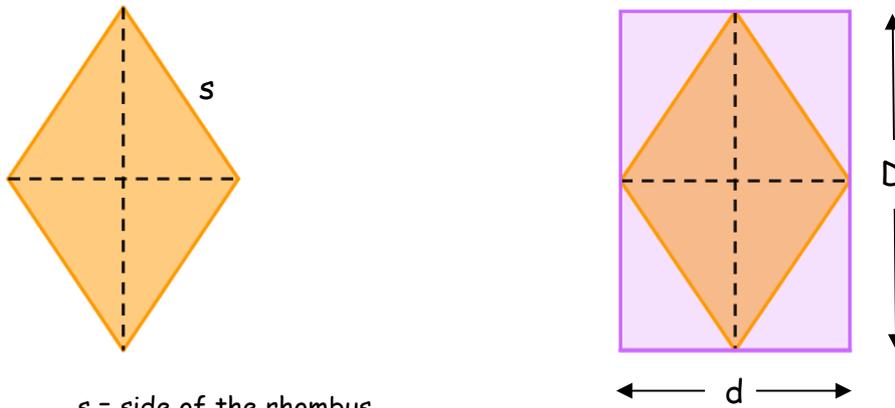
### Exercise 4:

The area of a square field is  $225 \text{ m}^2$ . Find the perimeter of the field.

### Exercise 5:

The perimeter of a rectangular carpet is 20 m. If the width of the carpet is 4 m, what is its length?

## Rhombus



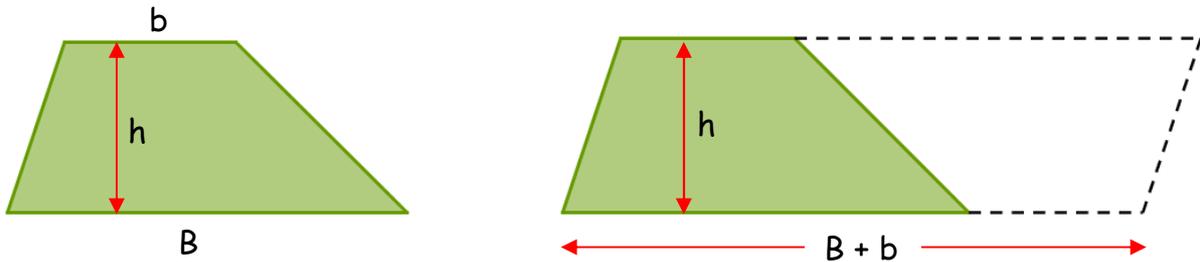
$s$  = side of the rhombus  
 $"D"$  and  $"d"$  are the diagonals of the rhombus

The area of the rhombus is one half of the area of the rectangle.

$$\text{Area} = \frac{D \cdot d}{2}$$

$$\text{Perimeter} = 4s$$

## Trapezium



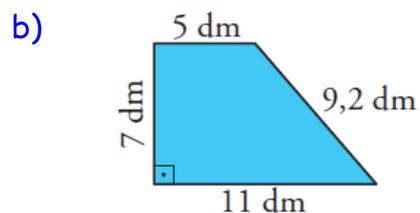
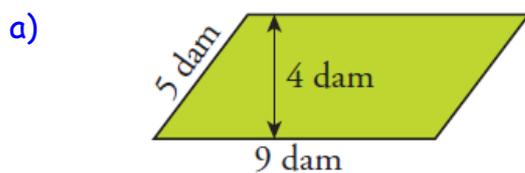
$h$  = height of the trapezium  
 $"B"$  and  $"b"$  are the bases of the trapezium

The area of the trapezium is one half of the area of the parallelogram whose base is  $"B + b"$  and whose height is  $"h"$ .

$$\text{Area} = \frac{(B + b) \cdot h}{2}$$

### Exercise 6:

Find the area and the perimeter of the following quadrilaterals.



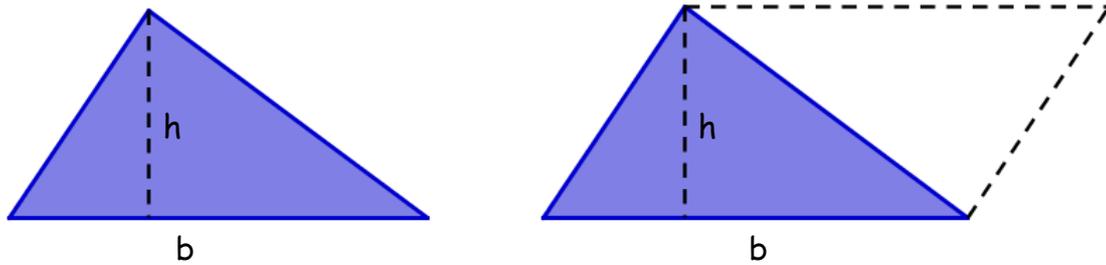
### Exercise 7:

The diagonals of a rhombus are 37 and 52 cm. Find its area.

### Exercise 8:

The diagonal of a square is 15 cm. Find its area.

## 12.3.- AREA OF A TRIANGLE

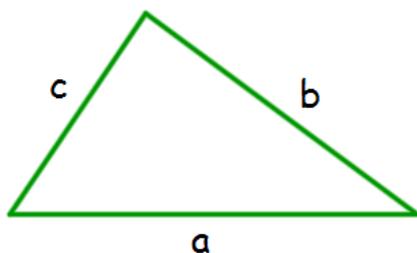


b = base of the triangle  
h = height of the triangle

The area of the triangle is one half of the area of the parallelogram.

$$\text{Area} = \frac{b \cdot h}{2}$$

You can also calculate the area of a triangle if you know the lengths of all three sides, using a formula that is called "**Heron's Formula**" after Heron of Alexandria.



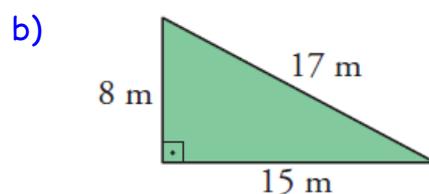
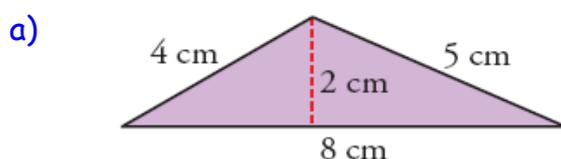
$$A = \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$$

$$s = \frac{a + b + c}{2}$$

(s = semiperimeter of the triangle)

### Exercise 9:

Find the area and the perimeter of the following triangles.



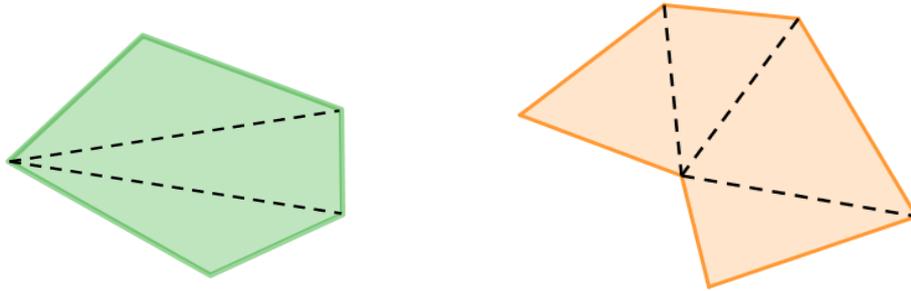
### Exercise 10:

The three sides of a right triangle are 18, 24 and 30 cm.

- Find the area of the triangle.
- Find the length of the height to the hypotenuse.

## 12.3.- AREA OF A POLYGON

Look at these irregular polygons.

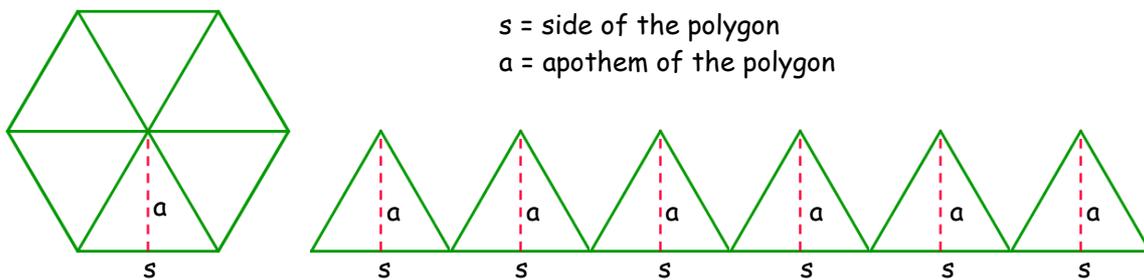


These polygons can be divided into triangles by drawing all the diagonals from one of the vertices. If you find the area of each triangle, then you can simply add them up to find the total area of the polygon.

### Area of a regular polygon

A regular polygon of  $n$  sides can be divided into  $n$  equal triangles.

Look at these regular hexagon:



Since the 6 triangles are equal, the area of the polygon will be six times the area of the triangle, that is

$$\text{Area} = 6 \cdot \frac{s \cdot a}{2} = \frac{6s \cdot a}{2} \quad 6s = \text{perimeter of the polygon}$$

In general, the area of a regular polygon is

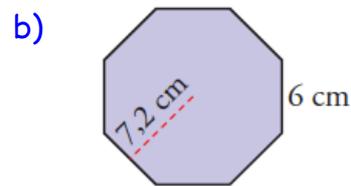
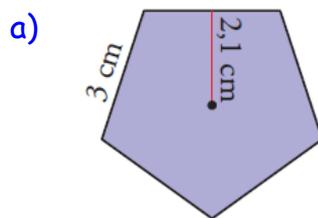
$$\text{Area} = \frac{\text{perimeter} \cdot \text{apothem}}{2}$$

**Exercise 11:**

The side of a regular octagon is 15 cm, and its apothem is 18 cm. Find its area.

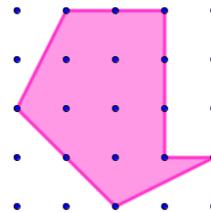
**Exercise 12:**

Find the area of the following regular polygons.



**Exercise 14:**

This shape is drawn on cm squared paper. Calculate the area of the shaded shape



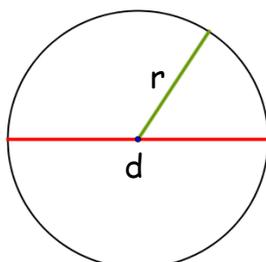
**Exercise 13:**

Use squared paper.

- a) Draw three different triangles with an area of 8 sq units.
- b) Draw three different parallelograms with an area of 12 sq units.
- c) Draw three different trapezia with an area of 12 sq units.

### 12.3.- AREAS AND LENGTHS ON THE CIRCUMFERENCE

The set of points that are the same distance from a fixed point is a **circumference**.



The **diameter**, *d*, is the distance across the circle through the centre.

The **radius**, *r*, is the distance from the centre to the edge.

The length of the circumference is in direct **proportion** to its diameter:  $C \approx 3d$

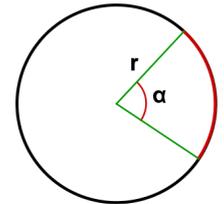
The actual proportion is a decimal. You use a symbol,  $\pi$  (pi).

$$L = \pi \cdot d \quad \text{or} \quad L = 2 \cdot \pi \cdot r$$

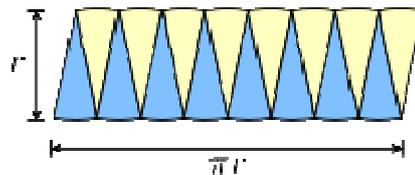
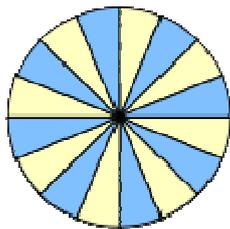
L: length of the circumference  
 d: diameter  
 r: radius  
 $\pi = 3.141592... \approx 3.14$

**Exercise 15:**

Prove that the length of an arc of a circumference that makes a central angle of  $\alpha$  degrees is  $l = 2 \cdot \pi \cdot r \cdot \frac{\alpha}{360}$ .



You can cut a **circle** into lots of sectors and lay them out side by side, to make an approximate parallelogram.



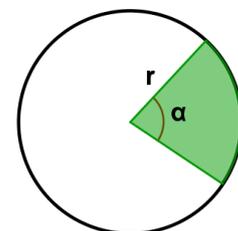
Area of the "parallelogram" = length · width =  $(\pi \cdot r) \cdot r = \pi \cdot r^2$

So the area of the circle is

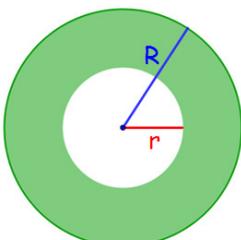
$$A = \pi \cdot r^2$$

**Exercise 16:**

Prove that the area of a circle sector that makes a central angle of  $\alpha$  degrees is  $A = \pi \cdot r^2 \cdot \frac{\alpha}{360}$ .



**Exercise 17:**



Prove that the area of an annulus is  $A = \pi \cdot (R^2 - r^2)$

**Exercise 18:**

A circular pond has radius 3 m. Work out the circumference of the pond.

**Exercise 19:**

A round hole has circumference of 44 cm. Work out the radius of the hole, to 1 decimal place.

**Exercise 20:**

A flowerbed in the park is semicircular. It has a radius of 2 m.



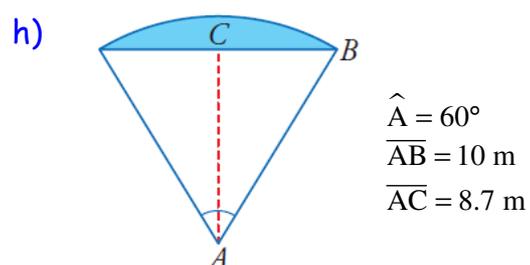
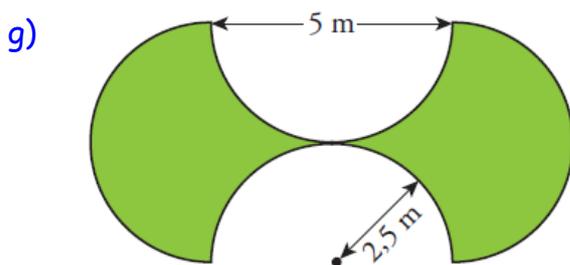
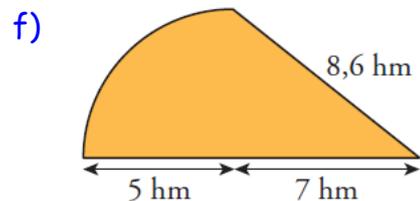
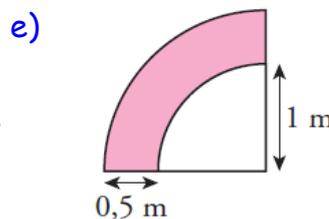
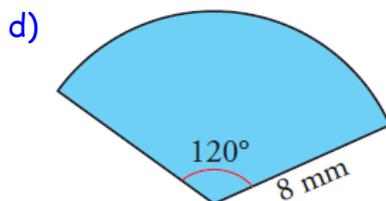
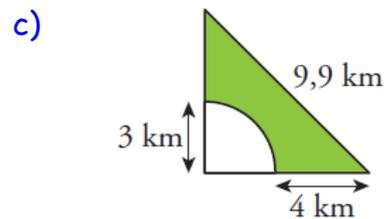
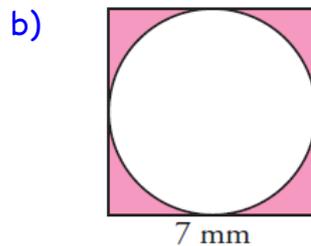
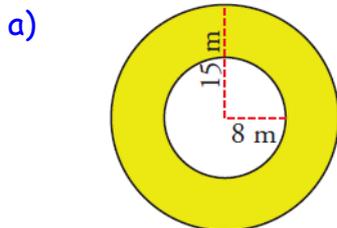
a) Work out the area of the flowerbed.

Percy the park keeper wants to plant flowers that each need an area of  $0.3 \text{ m}^2$ .

b) How many of these flowers can Percy plant in the flowerbed? What space does he have left?

**Exercise 21:**

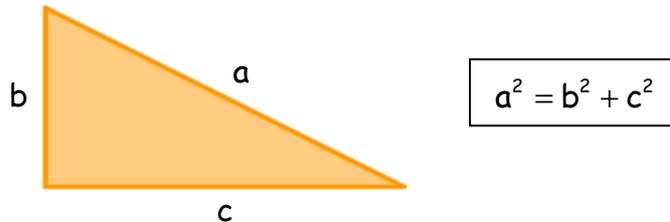
Find the area and the perimeter of the coloured shapes.



## 12.4.- PYTHAGORAS' THEOREM AND AREAS

Remember the Pythagora's theorem:

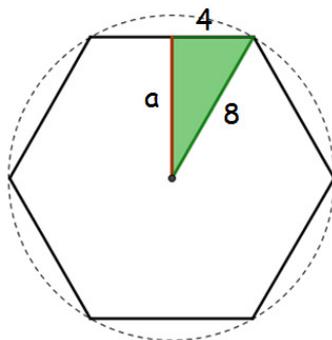
In a right angled triangle: the square of the hypotenuse is equal to the sum of the squares of the two legs.



Pythagoras' theorem allows us to find a missing side in right-angled triangles. This fact is very useful when we are dealing with problems that are related to the calculation of areas.

**Example:**

Find the area of a regular hexagon whose side is 8 cm.



The area of a regular polygon is

$$\text{Area} = \frac{\text{perimeter} \cdot \text{apothem}}{2}$$

In a regular polygon, the radius of the circumcircle, the apothem and half side make a right triangle.

In a regular hexagon the radius of the circumcircle and the side of the polygon have the same length.

Using Pythagora's theorem in the right triangle:

$$8^2 = a^2 + 4^2$$

$$64 = a^2 + 16 \Rightarrow 64 - 16 = a^2 \Rightarrow 18 = a^2 \Rightarrow a = \sqrt{18} \approx 6.9 \text{ cm}$$

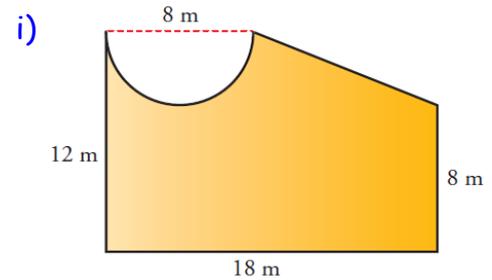
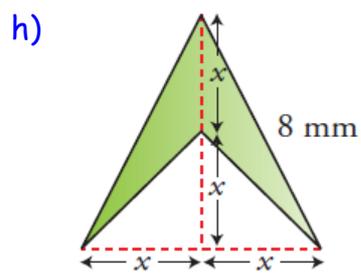
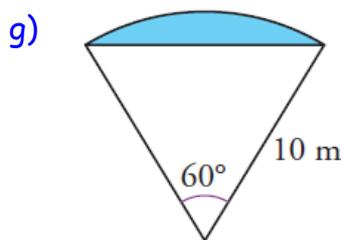
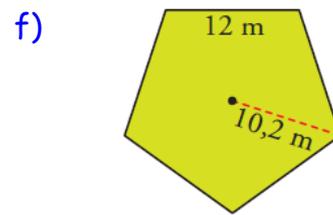
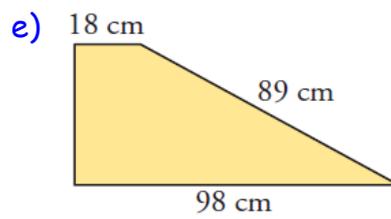
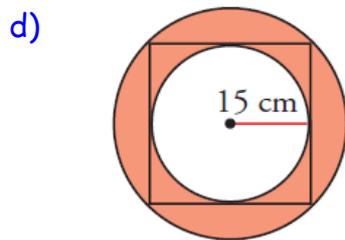
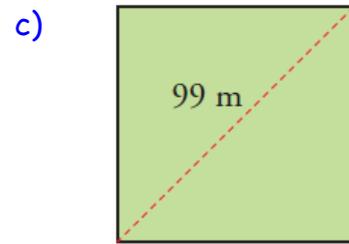
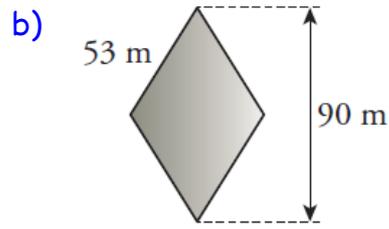
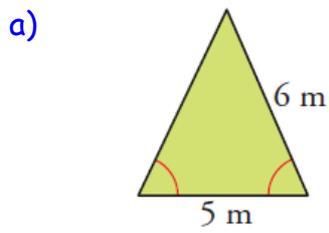
$$\text{Area} = \frac{\text{perimeter} \cdot \text{apothem}}{2} = \frac{6 \cdot 8 \cdot 6.9}{2} = \underline{\underline{165.6 \text{ cm}^2}}$$

**Exercise 22:**

Find the area of an equilateral triangle whose side is 15 cm.

**Exercise 23:**

Find the area and the perimeter of the coloured shapes.



**Exercise 24:**

Find the area and the perimeter of the following shapes.

