## CHRISTMAS TREE

The students of the IES Don Bosco are going to make a Christmas Tree using recycled materials. The idea is to make round cardboard *trays* and to fill them with cans (soft drinks, beer, …). The height of the tree is 216 cm.

The height of a can is 11.5 cm and the thickness of the trays is 5 mm. How many trays will they need to make this tree?

The radius of the top tray is 9.5 cm and the radius of the bottom tray is 69 cm. If the difference between the radiuses of two consecutive trays is always the same, what will be that difference and what will be the radius of each tray?
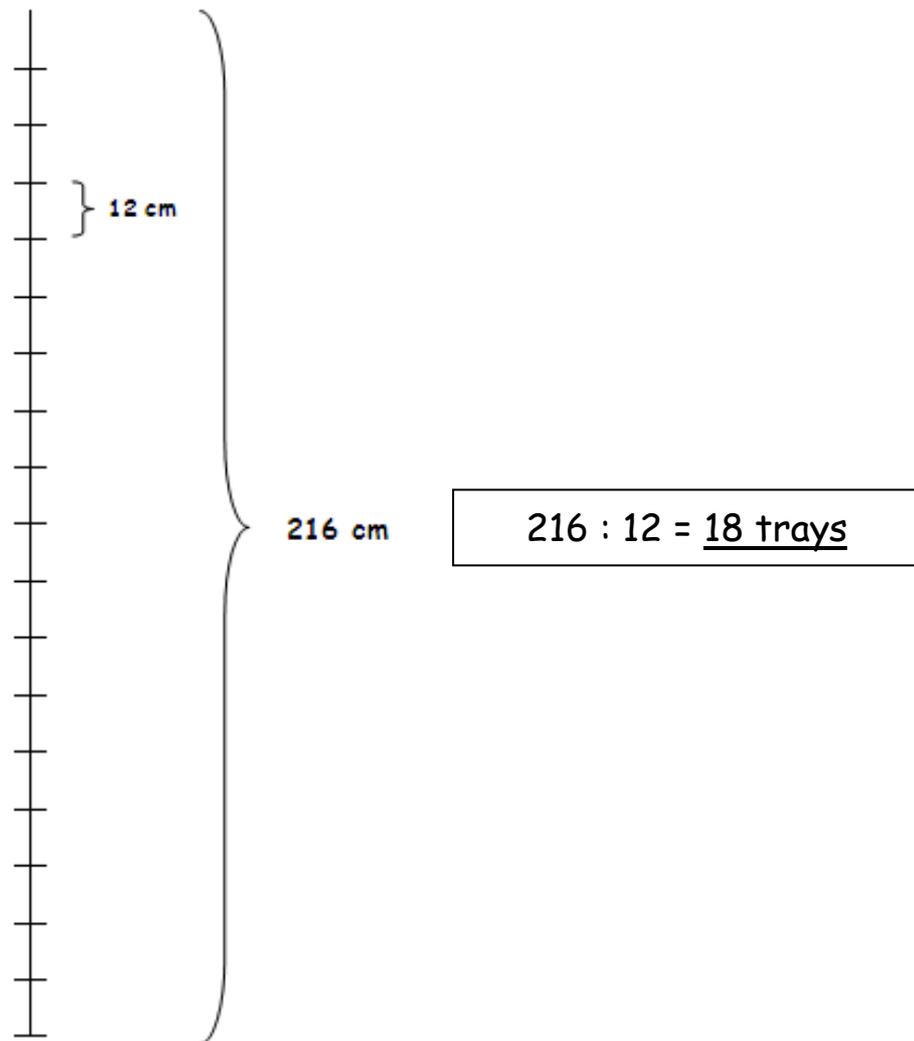
The sequence of these radiuses is a particular type of sequences. Could you find out the name of these sequences?
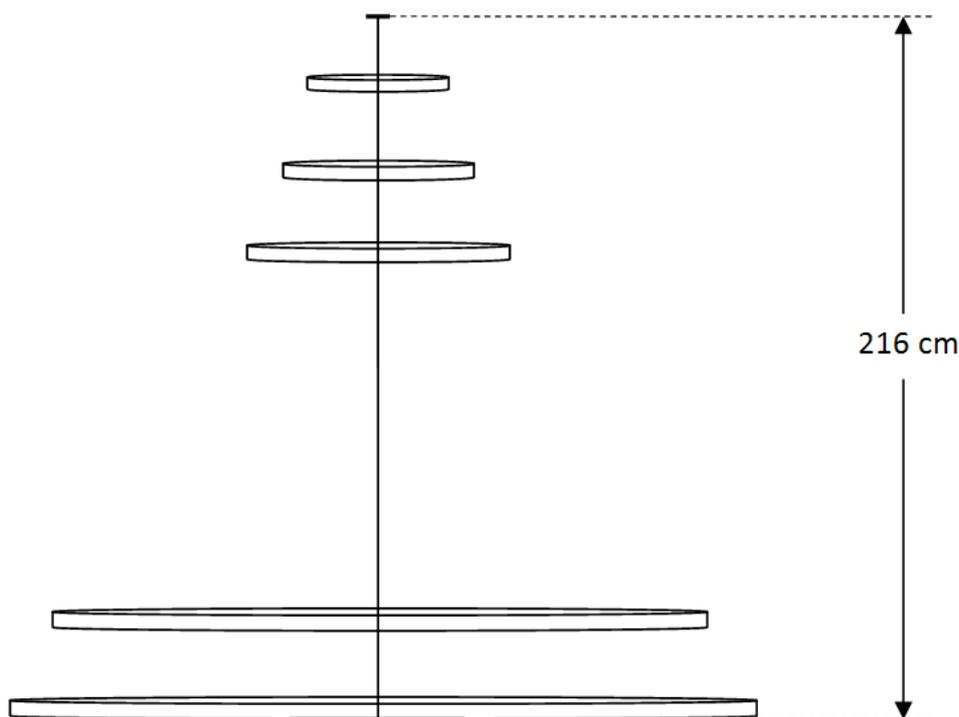
216 cm

<u>Answer</u>:

The height of a can is 11.5 cm and the thickness of a tray is 5 mm. If we add both distances, we get 11.5 + 0.5 = 12 cm.

The height of the tree is 216 cm. If we divide this height by 12 we will get the number of trays that we need to make the tree.

Let r_1, r_2, r_3, ... , r_{18} be the radiuses of the 18 trays.

We know that r_1 = 9.5 cm and r_{18} = 69 cm.

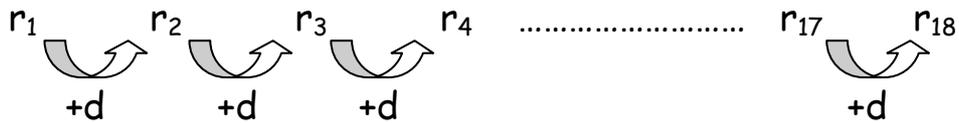The difference between two consecutive radiuses is the same.

Let "d" be this difference.

$$r_2 - r_1 = d \Rightarrow r_2 = r_1 + d$$

$$r_3 - r_2 = d \Rightarrow r_3 = r_2 + d$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$r_{18} - r_{17} = d \Rightarrow r_{18} = r_{17} + d$$

$r_1 \quad r_2 \quad r_3 \quad r_4 \quad \dots\dots\dots\dots\dots \quad r_{17} \quad r_{18}$

+d   +d   +d   +d

$$r_{18} - r_1 = 17d \Rightarrow 69 - 9.5 = 17d \Rightarrow 59.5 = 17d \Rightarrow d = \frac{59.5}{17} = 3.5 \text{ cm}$$

Therefore, the difference is 3.5 cm and the radiuses of the trays are:

r_1 = 9.5 cm	r_{10} = 41 cm
r_2 = 13 cm	r_{11} = 44.5 cm
r_3 = 16.5 cm	r_{12} = 48 cm
r_4 = 20 cm	r_{13} = 51.5 cm
r_5 = 23.5 cm	r_{14} = 55 cm
r_6 = 27 cm	r_{15} = 58.5 cm
r_7 = 30.5 cm	r_{16} = 62 cm
r_8 = 34 cm	r_{17} = 65.5 cm
r_9 = 37.5 cm	r_{18} = 69 cm

The sequences where the difference between two consecutive terms is the same are called **arithmetic sequences** or **linear sequences**.

Therefore, the sequence of the radiuses of the trays will be a linear sequence.